Network-wide resource optimization of wireless OFDMA mesh networks with multiple radios

Pablo Soldati
Royal Institute of Technology KTH,
SE-100 44 Stockholm, Sweden
Email: pablo.soldati@ee.kth.se

Mikael Johansson
Royal Institute of Technology KTH,
SE-100 44 Stockholm, Sweden
Email: mikael.johansson@ee.kth.se

***Abstract*— We consider the problem of joint end-to-end rate optimization and radio resource management in wireless OFDMA-based mesh networks. Radio units equipped with multiple radio interfaces are combined with the OFDMA medium access scheme to make up for the classical limitations of single-carrier wireless networks. We pose the problem as a utility maximization problem subject to link-rate constraints, power control and transmission scheduling in terms of time slots, channels and radio interfaces. The optimal solution to this problem is in general hard to achieve, both in theory and in practice. We propose an alternative solution approach that drastically reduces the computation time, and suggest a heuristic scheme that attempts to reduce the computation time even further while maintaining the overall system performance close to the optimal. For each scheme we validate the theoretical framework and quantify the performance with numerical examples.**

I. Introduction

High data-rate wireless networks have recently been proposed to replace wired networks in many applications. The IEEE 802.16 family of standards [1], often referred to as WiMax, has been conceived to provide high-speed wireless communications to a large number of users across wide areas. Standards-based devices will offer a mobile and quickly deployable alternative to the current cabled networks [2], combining communications in the frequency bands $10 - 66$ GHz and $2 - 11$ GHz with multiple access technologies that enable both single- and multi-carrier division of the available spectrum. Backhaul and local services are offered using point-to-multipoint or mesh topologies, where mesh provides the framework for multihop networking.

The multiple access technology strongly affects the system capacity. For a single-carrier scenario, the network capacity can be increased by allowing links to share the medium via spatial reuse of time slots [3], [4], [5], [6]. The division of the bandwidth in multiple carriers can be exploited using either Orthogonal Frequency Division Multiplexing (OFDM) or Orthogonal Frequency Division Multiple Access (OFDMA) schemes. OFDM is a spectrally efficient digital modulation technique that divides the system bandwidth into a number of narrowband overlapping sub-carriers so as to transform the frequency selective fading channel into several flat fading channels. Users transmit their data over all the subcarriers in order to exploit the channel diversity. Multiple access is obtained allowing users to transmit in different symbol periods. OFDMA is similar to OFDM in that it divides the bandwidth into sub-carriers, but it goes one step further by grouping multiple sub-carriers into sub-channels. A user might transmit using all the sub-carriers (as in OFDM), or multiple clients might transmit simultaneously using subsets of sub-carriers to exploit location-dependent channel diversity and frequency selectivity. Adaptive sub-carrier allocation combined with adaptive bit loading and power control for maximizing user throughput or minimizing energy consumption in OFDMA-based networks has been under the attention of several researchers [2], [7], [8]. However, to the best of our knowledge, no study has analyzed the effects of radio resource management on the end-to-end user requirements in OFDMA-based networks, *i.e.* how the link scheduling (time slot and sub-carrier assignment), power control and rate adaption schemes can be designed to optimize the end-to-end rates. The channel assignment problem in multi-channel multi-radio wireless mesh networks has been studied in [9], [10], [11], but only using simplistic interference models where the ability of two nodes to communicate reliably depends only on their distance. In contrast, we use a more realistic model where the link quality depends on the actual signal to interference and noise ratio (SINR).

Specifically, we consider the problem of assigning sub-carriers to wireless links in multihop mesh networks where nodes have the capability to use a maximum (fixed) number of radio interfaces. We exploit an SINR-based power-rate relationship and define the optimal network operation in terms of a utility maximization problem subject to link capacity constraints, power and rate control and time-frequency assignment. Spatial reuse of subcarriers is allowed to increase the capacity of the network. In order to mitigate the multiple access interference that arises due to simultaneous transmissions, power control is used to maintain the quality of active links so as to sustain a desired data rate in each subcarrier. Finding the optimal solution to this problem is computationally hard, and appears to be out of reach for commercial optimization solvers. To this end, we develop a specialized algorithm that allows to compute optimal and near-optimal solutions in reasonable time and suggest a fast heuristic that results in relatively modest performance losses. This allows us to compute the performance limits of multi-radio wireless OFDMA mesh networks and provides a benchmark for alternative (more practical) strategies.

II. Network Model

We consider a multiple carrier wireless mesh network with nodes located at fixed positions. Each node is assumed to have infinite buffering capacity and the ability to transmit, receive and relay data to other nodes using up to K_n radio interfaces.

A. Network topology and offered link rates

We represent the network topology by a directed graph, with nodes labelled $n = 1, \dots, N$, links labelled $l = 1, \dots, L$ and channels labelled $f = 1, \dots, F$. A link is represented by an ordered pair (i, j) of distinct nodes, where the presence of link (i, j) means that the network is able to send data from node i to node j. The network topology is described by a node-arc incidence matrix $\mathbf{A} \in \mathbb{R}^{N \times L}$, whose entry a_{nl} is 1 if link l is outgoing from node n, -1 if l is incoming to node n and 0 otherwise. Hence, for each node n we define $\mathcal{I}(n)$ and $\mathcal{O}(n)$ as the sets of incoming and outgoing links respectively.

The system bandwidth W is divided into a number F of equally sized and non-overlapping channels. When multiple links share the same channel to transmit, interference will occur. In each channel, a node can select a link to either transmit or receive data from (at most) one other node. To model this, let G_{lmf} be the effective link gain between the transmitter of link m and the receiver of link l when using channel f (including distance-based attenuation and fading as well as the effects of coding gain, spreading gain and beamforming, see *e.g.*, [12], [13]), let σ_{lf} be the thermal noise power at the receiver of link l and P_{lf} be the power used by its own transmitter. We assume that each transmitter is subject to a simple power limit $0 \leq P_{lf} \leq P_{\max}$, and we define the signal to noise and interference ratio of link l as

$$\gamma_{lf}(\mathbf{p}_f) = \frac{G_{llf} P_{lf}}{\sigma_{lf} + I_{lf}} \tag{1}$$

where $\mathbf{p}_f = \begin{pmatrix} P_{1f} & \cdots & P_{Lf} \end{pmatrix}^T$ denotes the channel power allocation vector, and $I_{lf} = \sum_{m \neq l} G_{lmf} P_{mf}$ is the interference experienced at the receiver of link l. The matrix $\mathbf{P} = [\mathbf{p}_1 \ \dots \ \mathbf{p}_F]$ summarizes the power allocation in the entire frequency spectrum. We consider each link as a single-user Gaussian channel with Shannon capacity $c_{lf}(\mathbf{p}_f) = \frac{W}{F} \log(1 + \gamma_{lf}(\mathbf{p}_f))$. We assume that a link is able to transmit reliably when its SINR is over a predefined threshold γ^{tgt}, which also defines a unique link rate $r^{\text{tgt}} = \frac{W}{F} \log(1 + \gamma^{\text{tgt}})$. To this end we introduce the following rate allocation policy

$$r_{lf} = \begin{cases} r^{\text{tgt}}, & \text{if} \quad \gamma_{lf}(\mathbf{p}_f) \geq \gamma^{\text{tgt}} \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Thus in each channel, a node n may transmit with link l at rate r^{tgt} if the SINR level at its receiver exceeds the threshold, and must stay silent otherwise. Since a node can simultaneously transmit on multiple channels, the total transmission rate experienced by a link l can be modelled as

$$r_l = \textstyle\sum_{f=1}^{F} r_{lf} x_{lf} \quad \forall l = 1, \dots, L \tag{3}$$

where $x_{lf} = 1$ if channel f is used by link l, and 0 otherwise. This leads to a finite number of feasible link-rate vectors

$$\mathbf{c}^{(k)} = (r_1 \dots r_L) \qquad k = 1, \dots, K$$

where r_l id given by (3). Although K may be as large as $(K_n + 1)^L$, most networks will, due to interference and other technological constraints, support substantially fewer link rate vectors. By time-sharing between a given set of link rate vectors $\{\mathbf{c}^{(k)} \mid k \in \mathcal{K}\}$, we can achieve the following polyhedral rate region

$$\mathcal{C}^{\mathcal{K}} = \left\{ \mathbf{c} = \sum_{k \in \mathcal{K}} \alpha_k \mathbf{c}^{(k)} \mid \alpha_k \geq 0, \ \sum_{k \in \mathcal{K}} \alpha_k = 1 \right\}$$

Here, the time-sharing coefficients α_k represent the fraction of schedule length in which rate vector $\mathbf{c}^{(k)}$ is activated. If $\mathcal{C}^{\mathcal{K}}$ contains all feasible rate-vectors, we will simply drop the superscript and use the short-hand notation $\mathbf{c} \in \mathcal{C}$ to denote that $\mathbf{c}$ is an achievable long-term average link rate. The allocation of time slots and sub-carriers to links can be represented as in Figure 1.

B. End-to-end rates, routing and link-rate constraints

Nodes are assumed to always have data to send to the other nodes, possibly via multi-hop routing. We label the source-destination pairs by integers $p = 1, \dots, P$ and let s_p denote the end-to-end rate for communication between source-destination pair p. Associated with each pair p is a utility function $u_p(\cdot)$, which describes the utility of the pair to communicate at rate s_p (cf. [14]). We assume that u_p is increasing and strictly concave, with $u_p \to -\infty$ as $s_p \to 0^+$. Each demand is assumed to be routed along a single fixed path between source and destination. The routing is specified by a routing matrix $\mathbf{R} = [r_{lp}] \in \mathbb{R}^{L \times P}$ where

$$r_{lp} = \begin{cases} 1 & \text{if data between pair } p \text{ is routed across link } l \\ 0 & \text{otherwise} \end{cases}$$

Letting c_l denote the transmission rate of link l, $\mathbf{c} = [c_l]$ be the vector of link-capacities, the vector of total traffic across links is given by $\mathbf{Rs}$ and the network flow model imposes the following set of constraints on the end-to-end rate vector $\mathbf{s}$

$$\mathbf{Rs} \preceq \mathbf{c} \qquad\qquad \mathbf{s} \succeq \mathbf{0}$$

where the link rates depend on the medium access scheme, channel conditions and the allocation of radio resources, such as channel, transmit powers and time-slots, to the transmitters.

III. Cross-Layer optimization

In the past decade, many cross-layer optimization schemes have been proposed to jointly coordinate end-to-end data flows over TCP/IP wired networks. The basic intuition proposed in [14], [15] states that the optimal network operation can be modelled as the utility maximization problem

$$\begin{array}{ll} \text{maximize} & \sum_p u_p(s_p) \\ \text{subject to} & \mathbf{Rs} \preceq \mathbf{c} \\ & \mathbf{s} \succeq \mathbf{0} \end{array} \tag{4}$$

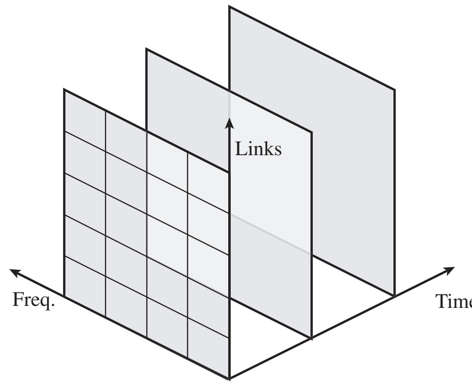

Fig. 1. Model of time and frequency for link scheduling.

where the end-to-end transmission rate vector $\mathbf{s}$ is optimized to maximize an aggregate network utility function, while the link capacity vector $\mathbf{c}$ is fixed. If the utility functions are logarithmic, the problem yields a proportionally fair allocation of end-to-end bandwidth [14]. A distributed solution to this problem can be derived using mathematical decomposition techniques and suggests simple resource allocation mechanisms implemented in end-hosts and routers [14].

When optimizing the operation of wireless networks it is crucial to notice that the links capacities are not fixed a priori, but depend in a non-trivial way on the allocation of radio resources at the physical and link layers. Aware of this fact, the utility maximization problem (4) can be reformulated as

$$\begin{array}{ll} \text{maximize} & \sum_p u_p(s_p) \\ \text{subject to} & \mathbf{R}\mathbf{s} \preceq \mathbf{c} \\ & \mathbf{c} \in \mathcal{C} \qquad \mathbf{s} \succeq \mathbf{0} \end{array} \tag{5}$$

where the optimization variables are the end-to-end data rates $\mathbf{s}$ and the links capacity vector $\mathbf{c}$.

A. Centralized solutions via column generation

A centralized solution to (5) based on column generation techniques has been proposed in [13] for STDMA wireless networks. A distributed solution to proportionally fair allocation of end-to-end rates in STDMA wireless networks has been developed in [16], while distributed solutions for scheduling and resource allocation in WiMax single-carrier networks are derived in [6]. The focus of this paper is to extend the analysis in [6] to the case of multi-channel multi-radio mesh networks. This includes a mathematical formulation of the problem within the framework of mixed-integer-programming, the development solution procedures based on multi-level column generation that drastically reduce the computation times, and the introduction of fast heuristics for suboptimal solutions, possibly amendable for a distributed implementation.

The basic idea of the column generation method is to sequentially compute lower and upper bounds to the network utility by restricting the optimization problem (5) to a subset of link-rate vectors $\mathcal{C}^{\mathcal{K}} \subseteq \mathcal{C}$, and to generate new link-rate vectors so that to guarantee the convergence of the lower and upper bounds. The lower bound is computed by optimizing the end-to-end rates $\mathbf{s}$ and the time-sharing coefficients $\boldsymbol{\alpha}$ for the current set of link-rate vectors. The upper bound is computed via duality: let $\boldsymbol{\lambda}$ be the vector of Lagrange multipliers for the capacity constraints $\mathbf{R}\mathbf{s} \preceq \mathbf{c}$ in the lower-bound computation and consider the Lagrangian

$$L(\mathbf{s}, \mathbf{c}, \boldsymbol{\lambda}) = \sum_p u_p(s_p) - q_p s_p + \sum_l \lambda_l c_l$$

where $q_p = \sum_l r_{lp} \lambda_l$. An upper bound to the network utility can be computed by maximizing $L(\mathbf{s}, \mathbf{c}, \boldsymbol{\lambda})$. This problem decomposes into two subproblems: a *scheduling subproblem*

$$\begin{array}{ll} \text{maximize} & \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{c} \\ \text{subject to} & \mathbf{c} \in \mathcal{C} \end{array} \tag{6}$$

and a *network subproblem*

$$\begin{array}{ll} \text{maximize} & \sum_p u_p(s_p) - q_p s_p \\ \text{subject to} & s_p \geq 0 \end{array} \tag{7}$$

By adding the optimal solution $\mathbf{c}^\star \in \mathcal{C}$ of (6) to the set of link-rate vectors (*i.e.*, letting $\mathcal{C}^{\mathcal{K}} = \mathcal{C}^{\mathcal{K}} \cup \mathbf{c}^\star$) and repeating the optimization one can guarantee convergence to the optimal solution in a finite number of iterations (see [13] for details).

An interesting aspect of the column generation procedure (and dual decomposition approaches in general) is that the decomposition into network and scheduling subproblems (6)-(7) can be applied irrespectively of radio technology. The end-to-end flow control formulation is invariant to lower layer details, while the scheduling subproblem (and its solution complexity) varies with the underlying radio technologies. In the rest of the paper we will focus on solving the scheduling subproblem for mesh network with multiple channels and user with multiple radio interfaces.

B. The multi-radio multi-channel scheduling subproblem

Unlike the solution proposed in [6], the opportunity of nodes to use multiple radio interfaces combined with the multiple channels offered by the OFDMA modulation scheme makes the computational cost of the scheduling subproblem (6) rather high, already for relatively small networks. In order to model the multi-radio capabilities of a node we define two matrices $\mathbf{X} \in \{0,\ 1\}^{L \times F}$ and $\mathbf{Y} \in \{0,\ 1\}^{N \times F}$ whose entries are

$$x_{lf} = \begin{cases} 1 & \text{if link } l \text{ transmits over channel } f \\ 0 & \text{otherwise} \end{cases}$$

$$y_{nf} = \begin{cases} 1 & \text{if node } n \text{ transmits or receives over channel } f \\ 0 & \text{otherwise} \end{cases}$$

In each channel, a node can either transmit or receive data from at most one other node, $i.e.$

$$\sum_{l \in I(n) \cup O(n)} x_{lf} \leq y_{nf} \quad n = 1, \ldots, N \quad f = 1, \ldots, F \tag{8}$$

Moreover, in each time slot a node is allowed to transmit using up to K_n radio interfaces, hence

$$\sum_{f=1}^{F} y_{nf} \leq K_n \quad n = 1, \ldots, N \tag{9}$$

In each channel, a feasible transmission group is defined as a set of links and an associated power allocation $\mathbf{p}_f$ such that the signal to interference and noise ratios of the active links exceed the target. In other words, active links must satisfy

$$\frac{G_{llf}P_{lf}}{\sigma_{lf}+\sum_{m\neq l} G_{lmf}P_{mf}} \geq \gamma^{\text{tgt}}$$

According to [5], the above constraint can be written as

$$G_{llf}P_{lf} + (1-x_{lf})M_{lf} \geq \gamma^{\text{tgt}}\left(\sigma_{lf}+\sum_{m\neq l} G_{lmf}P_{mf}\right) \tag{10}$$

for a sufficiently large constant M_{lf}. We choose $M_{lf} = \gamma^{\text{tgt}}(\sigma_{lf}+\sum_{m\neq l} G_{lmf}P_{\max})$ and rewrite (10) for all f and l in a more compact form as

$$\mathbf{A}_x\mathbf{x} + \mathbf{A}_p\mathbf{p} \preceq \mathbf{b} \tag{11}$$

where the vectors $\mathbf{x} = \text{vec}(\mathbf{X})$ [1] and $\mathbf{p} = \text{vec}(\mathbf{P})$ describe the link scheduling and power allocation respectively, while the matrices $\mathbf{A}_x$, $\mathbf{A}_p$ and the vector $\mathbf{b}$ model the constraints (10) for all links in all frequencies.

The multi-radio multi-channel scheduling problem (6) can then be written as the mixed-integer linear program (MILP)

$$\begin{array}{ll} \text{maximize} & \sum_l \lambda_l \sum_f x_{lf} r^{\text{tgt}} \\ \text{subject to} & \mathbf{A}_x\mathbf{x} + \mathbf{A}_p\mathbf{p} \preceq \mathbf{b} \\ & (8),\ (9) \\ & 0 \leq P_{lf} \leq P_{\max} \quad \forall l, f \\ & x_{lf} \in \{0,\ 1\} \quad \forall l, f \\ & y_{nf} \in \{0,\ 1\} \quad \forall n, f \end{array} \tag{12}$$

C. Efficient frequency assignment via column generation

The MILP (12) jointly optimizes the link scheduling and power allocation over F channels, and needs to be solved at each iteration of the column generation in order to find a new link-rate vector $\mathbf{c}$ to add to the set $\mathcal{C}^{\mathcal{K}}$. Unfortunately, the computation times for solving the scheduling subproblem grows very quickly with the number of links and channels, and it is hard to get computational results in reasonable time even with state-of-the art mathematical programming software.

In [17], the author has proposed to solve resource allocation problems of a similar type to (12) by a variation of a column generation technique proposed by Mehrota and Trick [18]. Although the aim in [17] is to minimize the schedule length in an STDMA network, it can be adapted to our case where we have a maximum schedule length of F transmission groups, *i.e.* one for each channel. To this end, let $\mathcal{T} = \{\mathbf{t}_k |\ \mathbf{t}_k \in \{0,\ 1\}^L,\ k = 1,\ldots,K\}$ be the set of feasible transmission groups (*i.e.*, satisfying (10) and the power constraints), define $\widetilde{\boldsymbol{\lambda}} = \boldsymbol{\lambda} r^{\text{tgt}}$ and $\delta_k = \widetilde{\boldsymbol{\lambda}}^T \mathbf{t}_k$ and rewrite (12) as

$$\begin{array}{ll} \text{maximize} & \sum_k \delta_k z_k \\ \text{subject to} & \mathbf{W}\mathbf{z} \preceq \mathbf{1}K_n \\ & \sum_k z_k \leq F \quad z_k \in \{0,\ 1\} \end{array} \tag{13}$$

[1] We define $\text{vec}(X) \triangleq [x_{11},\ldots,x_{L1},x_{12},\ldots,x_{L2},\ldots,x_{1F},\ldots,x_{LF}]^T$.

where $\mathbf{W} = [\mathbf{w}_1 \ldots \mathbf{w}_K]$ with $\mathbf{w}_k = |\mathbf{A}|\mathbf{t}_k$ and $z_k \in \{0,1\}$. The first set of constraints ensures that at most K_n radio interfaces are used at each node, while the second set ensures that at most F feasible transmission groups are allocated.

Let $\mathcal{T}^{\mathcal{K}} \subseteq \mathcal{T}$ be a subset of transmission groups and consider the LP relaxation of the associated restriction of (13)

$$\begin{array}{ll} \text{maximize} & \sum_{k\in\mathcal{T}^{\mathcal{K}}} \delta_k z_k \\ \text{subject to} & \mathbf{W}\mathbf{z} \preceq \mathbf{1}K_n \\ & \sum_{k\ \in\mathcal{T}^{\mathcal{K}}} z_k \leq F \quad z_k \in [0,\ 1] \end{array} \tag{14}$$

When this master problem has been solved, we need to identify if the objective function can be improved by adding a new transmission group. Let $\boldsymbol{\mu}$ be the Lagrange multiplier vector for the constraints $\mathbf{W}\mathbf{z} \preceq \mathbf{1}K_n$ and consider the Lagrangian

$$L(\mathbf{z},\mu) = \sum_{k\in\mathcal{T}^{\mathcal{K}}} \delta_k z_k - \mu^T(\mathbf{W}\mathbf{z} - \mathbf{1}K_n) = \tag{15}$$

$$= \sum_{k\in\mathcal{T}^{\mathcal{K}}} (\widetilde{\lambda}^T - \mu^T|\mathbf{A}|)\mathbf{t}_k z_k + \mu^T\mathbf{1}K_n \tag{16}$$

When the master problem may be improved, it is natural to add the transmission group that solves

$$\begin{array}{ll} \text{maximize} & (\widetilde{\lambda}^T - \mu^T|\mathbf{A}|)\mathbf{t} \\ \text{subject to} & (10),\ 0 \leq P_{lf} \leq P_{\max} \quad \forall l, f \\ & \mathbf{t} \in \{0,\ 1\}^L \end{array} \tag{17}$$

and repeat the master problem. The column generation procedure for solving the scheduling subproblem (6) then follows the same principles as the column generation for the overall problem (5) (see Section III). The procedure is summarized as Algorithm 1. Upon termination, a feasible channel allocation is computed by solving (14) with an integer restriction on the z-variables. Although this integer solution is not necessarily optimal, it has turned out to be optimal or near-optimal for all cases we have considered. Optimality can be enforced by implementing the full branch-and-price procedure described in [18]. To generate provably optimal solutions, we have used the result of the column generation to warm-start a commercial branch-and-bound solver. The branch-and-bound solver is often able to prove optimality within a very small number of iterations.

The overall solution we propose can be considered as a multi-level column generation procedure: on the outer level, we optimize end-to-end rates and time-slot allocations. Each iteration of this procedure requires the solution of a scheduling subproblem across sub-carriers and transmit powers which, in turn, is solved by a column generation technique.

IV. A GREEDY HEURISTIC FOR CHANNEL ALLOCATION

Although the technique proposed in Section III-B improves the overall performance of the cross layer optimization compared to the direct use of an integer programming solver, both solutions remain far from being suitable for real applications due to the considerable computational effort they require.

As an alternative, we propose a heuristic approach that solves the link scheduling problem separately for each channel

Algorithm 1 Subcarrier allocation via column generation

1: let $\mathcal{T}^0$ be an initial feasible channel allocation, ϵ be a small constant and $k = 0$.
2: **loop**
3: $k = k + 1$
4: solve (14) to get a lower bound u_{lower} of the optimal solution of problem (13).
5: solve (17) to get a new feasible channel allocation $\mathbf{t}_k$ and use (16) to obtain an upper bound u_{upper}.
6: $\mathcal{T}^{\mathcal{K}} = \mathcal{T}^{\mathcal{K}} \cup \{\mathbf{t}_k\}$
7: **end loop** if $u_{\text{upper}} - u_{\text{lower}} \le \epsilon$
8: use $\mathcal{T}^{\mathcal{K}}$ to solve (14) with integer restrictions on z_k to obtain a feasible channel allocation for problem (6).

Algorithm 2 Greedy channel allocation

1: let $f = f_0$, $i = 0$, $\mathbf{c}^{(k)} = \mathbf{0}$, $\mathcal{N} = \{1, \ldots, N\}$, $\widetilde{\mathcal{N}} = \{\emptyset\}$.
2: **loop**
3: solve (18) to obtain link-rate vector $\mathbf{c}_f$ for channel f
4: $\mathbf{c}^{(k)} = \mathbf{c}^{(k)} + \mathbf{c}_f$.
5: for all $n \in \mathcal{N}$ update the number of allocated radios. If n has reserved K_n interfaces, move n from $\mathcal{N}$ to $\widetilde{\mathcal{N}}$.
6: load new frequency: $i = i + 1$, $f = f_i$
7: **end loop** if either $f = F$ or $\mathcal{N} = \{\emptyset\}$.

f. The single channel assignment problem can be formulated as follows:

$$\begin{array}{lll} \text{maximize} & \sum_l \lambda_l x_{lf} c_{\text{tgt}} & \\ \text{subject to} & \mathbf{A}_{xf}\mathbf{x}_f + \mathbf{A}_{pf}\mathbf{p}_f \preceq \mathbf{b}_f & \\ & \sum_{l \in I(n) \cup O(n)} x_{lf} \le 1 & \\ & 0 \le P_{lf} \le P_{\max} & l = 1, \ldots, L \\ & \mathbf{x}_f \in \{0,\ 1\}^L & \end{array} \tag{18}$$

where the vector $\mathbf{x}_f$ corresponds to the f^{th} column of the matrix $\mathbf{X}$, while $\mathbf{A}_{xf}$, $\mathbf{A}_{pf}$ and $\mathbf{b}_f$ summarize the SINR-based constraints of type (10) for the link scheduling on channel f.

To enforce the limitation on the number of radio interfaces per node, we solve the the assignment problem (18) sequentially for each channel until either all channels have been allocated or some nodes have assigned all their available radios. When a node has assigned all its radios, it is removed from the network and the procedure continues on the reduced network topology. This algorithm terminates when all channels are assigned or all nodes have used their radios, see Algorithm 2.

Note that while the column generation procedure attempts to compute a price for using the radios in each node (formally, the Lagrange multiplier vector $\boldsymbol{\mu}$) our greedy heuristic effectively sets the price to infinity when a node has been assigned all its radios. It is possible that better heuristics can be derived using better pricing schemes, but such investigations are out of the scope of the current paper.

V. Numerical examples

We can now evaluate the performance of the three approaches for solving the joint end-to-end rate optimization, scheduling, rate and power adaption scheme in multi-channel multi-radio wireless mesh networks proposed in the previous sections. To achieve proportional fairness we will assume logarithmic utility functions throughout. We will isolate and quantify the performance of the optimal solutions, the scheduling subproblem solved using an inner column generation, and the heuristic approach in terms of total aggregate utility and computation time.

A. Results for a sample network

We will show in detail the results achieved for a network consisting of 8 nodes and 36 links. The network was generated using a modified version of the procedure described in [5] adapted to meet the requirement of the IEEE 802.16 standard [1]. In particular, we investigate the ISM frequency band $2.4000 - 2.4835$ GHz splitting the system bandwidth in $F = 10$ equally sized sub-carriers, while each node is equipped with $K_n = 4$ radios. For each link, let the maximum transmission power be $P_{\max} = 100$mW, the path loss exponent $\alpha = 3.5$ and the thermal noise at the receivers $\sigma = 3.34 \times 10^{-12}$. We use a single SNR-target $\gamma^{\text{tgt}} = 10$ and, for simplicity, we use the Shannon capacity formula $r^{\text{tgt}} = \frac{W}{F}\log_2(1 + \gamma^{\text{tgt}})$ to relate rate to the SNR-target.

The results obtained for this particular network are shown in Figure 2 and are summarized in Tables I and II. The computation time required from the optimal solution exceeds 24 hours, whereas the solution proposed in section Section III-B approaches the optimal solution in a reasonable time (about 17 minutes). Moreover, the greedy channel allocation described in Section IV drastically reduces the computation time to only one minute leading to a performance loss of about 4.6% with respect to the optimal solution.

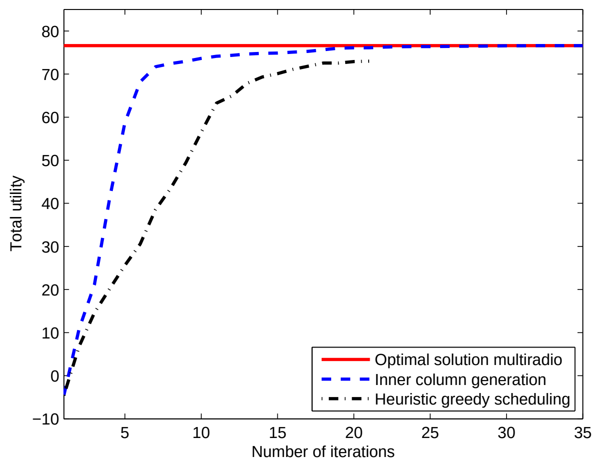

Fig. 2. Performance in terms of total aggregate utility.

B. Experience from a larger set of networks

We have evaluated the performance of the three variants of the joint congestion control, scheduling, rate and power adaption scheme for a set of randomly generated 10-node

	Total utility	%
Optimal solution	76.5891	$--$
Inner column gen.	76.5889	$-1.82 \cdot 10^{-4}$
Greedy Heuristic sched.	73.0052	-4.68

TABLE I
PERFORMANCE IN TERMS OF TOTAL AGGREGATE UTILITY.

	Hours	Minutes	Seconds
Optimal solution	$24+$	$--$	$--$
Inner column gen.	$--$	17	12
Greedy Heuristic sched.	$--$	1	2

TABLE II
PERFORMANCE IN TERMS OF COMPUTATION TIME.

multi-channel multi-radio wireless mesh networks. In this case, the performance loss of the inner column generation is within $1 - 2 \cdot 10^{-4}\%$ of the optimal performance and a computation time within $15 - 20$ minutes. The greedy channel allocation came within $3 - 12\%$ of the optimal performance, with an average performance loss of 8.4% and a computation time of $1 - 2$ minutes.

VI. CONCLUSIONS

We have considered the problem of joint end-to-end rate optimization and radio resource management in OFDMA mesh networks with multiple radios. We have posed the problem as a utility maximization problem subject to link-rate constraints, power control and transmission scheduling both in terms of time slot and channel allocation. Due to the computational complexity of the optimal solution, we have developed an alternative solution approach that achieve a near-optimal performance reducing at the same the computation effort. This methods allow us to solve problem instances that are outside the reach of commercial mathematical programming solvers. Moreover, we have proposed a heuristic scheme that attempts to reduce the computation time even further while maintaining the overall system performance close to the optimal. Simulation results have shown that the first scheme achieves a near-optimal (less than $2 \cdot 10^{-4}\%$ of performance loss) solution while reducing the computation time from days to $15 - 20$ minutes, whereas the heuristic reduces the computation time to the order of a few minutes with a reasonable small price in terms of performance loss.

There are several natural extensions to this work. First, the problem (12) might be modified to account for total power constraint for each node. Second, one might investigate distributed (sub)optimal solution approaches where nodes explore the link quality in the available sub-carriers and select a channel allocation negotiating with a group of neighboring nodes. Finally, a more complex channel model should be analyzed in order to better capture the effect of the OFDMA modulation scheme on the channel features and link assignment.

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