Cross-Layer Optimization of Wireless Networks Using Nonlinear Column Generation

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***Abstract*— We consider the problem of finding the jointly optimal end-to-end communication rates, routing, power allocation and transmission scheduling for wireless networks. In particular, we focus on finding the resource allocation that achieves fair end-to-end communication rates. Using realistic models of several rate and power adaption schemes, we show how this cross-layer optimization problem can be formulated as a nonlinear mathematical program. We develop a specialized solution method, based on a nonlinear column generation technique, and prove that it converges to the globally optimal solution. We present computational results from a large set of networks and discuss the insight that can be gained about the influence of power control, spatial reuse, routing strategies and variable transmission rates on network performance.**

***Index Terms*— Wireless networks, capacity region, fairness, ad hoc, optimization methods, multihop, routing, power control, scheduling, cross-layer protocol design.**

I. Introduction

Wireless ad hoc networks is a promising access technology for realizing the vision of ubiquitous communications. Such systems could allow rapid deployment with little planning or user-interaction and possibly coexist with a sparse fixed infrastructure. However, the design of radio resource management schemes that work reliably and efficiently in such a distributed and heterogeneous environment is a major engineering challenge. Despite the complexity of establishing the information-theoretic capacity of wireless networks, recent contributions have derived asymptotic bounds on their performance (see *e.g.*, [1], [2]) and established achievable rate regions for (small) ad hoc networks under variable transmission strategies [3]. In addition, several approaches for maximizing the throughput of wireless networks have been developed (see, *e.g.*, [4]–[8]).

Fairness is a key consideration in allocating limited resources within a network. It has been observed that maximizing the throughput of wireless networks can lead to grossly unfair communication rates between source-destination pairs [9]. To operate a wireless network in a fair and efficient way, it is thus important to have methods that allow us to find fair resource allocations and help us to understand the trade-offs between fairness and throughput. In this paper, we extend our previous work on simultaneous routing and resource allocation [4], [8] to joint scheduling, routing and power control for fair allocation of network resources. For a given network configuration, our approach provides the optimal cross-layer coordinated operation of transport, routing and radio link layers under several important rate and power adaption schemes. This allows us to gain insight in the influence of power control, spatial reuse, routing strategies and variable transmission rates on the network performance, and provides a benchmark for alternative (heuristic) strategies.

The research on optimal scheduling of transmissions in multihop radio networks has a long history (see, *e.g.*, [10]–[13]), and our effort is closely related to the recent work reported in [3], [5], [7]. Our contribution extends the previous approaches by allowing nonlinear performance objectives (necessary to obtain proportional fairness between connections) and multipath routing, and by incorporating several important rate and power adaption schemes from the wireless networking literature. The approach is based on a nonlinear column-generation method and generates a sequence of feasible resource allocations that converges to the optimum in a finite number of steps.

The paper is organized as follows: In Section II, we describe our mathematical model of the wireless network. We formulate the simultaneous routing, resource allocation and scheduling problem in Section III and develop a specialized column-generation algorithm for solving the optimization problem in Section IV. In Section V, we use the approach to investigate the benefits of various network configurations in terms of throughput and fairness. Final remarks and conclusions are collected in Section VI.

II. Model and Assumptions

We consider a communication network formed by a set of nodes located at fixed positions. Each node is assumed to have infinite buffering capacity and can transmit, receive and relay data to other nodes across wireless links. The network performance then depends on the interplay between end-to-end rate selection, routing, power and rate adaption and transmission scheduling. A model for these dependencies will be presented next.

A. Network flow model

Network topology: We represent the topology of the network by a directed graph, with nodes labelled $n = 1, \dots, N$ and links labelled $l = 1, \dots, L$. A link is represented by an

Manuscript received December 22, 2003; revised July 15, 2004 and December 17, 2004; accepted January 14, 2005. The associate editor coordinating the review of this letter and approving it for publication was D. Gesbert. This research was sponsored in part by the Swedish Science Foundation (VR), NSF grant No. 0140700, AFOSR grant F49620-01-1-0365, and DARPA contract MDA972-02-1-0004.

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Digital Object Identifier 10.1109/TWC.2006.02023.

ordered pair (i,j) of distinct nodes. The presence of link (i,j) means that the network is able to send data from the start node i to the end node j. The network topology can be represented by a *node-link incidence matrix* $A \in \mathbb{R}^{N \times L}$, whose entries a_{nl} satisfy $a_{nl} = 1$ if n is the start node of link l, $a_{nl} = -1$ is n is the end node of link l, and $a_{nl} = 0$ otherwise. We define $\mathcal{O}(n)$ as the set of links that are outgoing from node n, and $\mathcal{I}(n)$ the set of links that are incoming to node n.

Multicommodity network flows: We use a multicommodity flow model for describing multipath routing of packets across the network (cf., [4]). In this model, each node can send data to many destinations and receive data from many sources, but multicast is not considered. We assume that data flows are lossless across links and that the traffic flow can be split arbitrarily at nodes as long as the flow conservation law is satisfied. We identify the flows by their destinations, *i.e.*, flows with the same destination are considered as a single commodity regardless of their sources. We assume that the destination nodes are labelled $d = 1, \dots, D$ where $D \leq N$. For each destination d, we define a *source-sink vector* $s^{(d)} \in \mathbb{R}^N$ whose nth ($n \neq d$) entry, $s_n^{(d)}$, denotes the non-negative data rate injected to the network at node n (the source) destined for node d (the sink). In light of the flow conservation law, the sink flow at the destination is given by $s_d^{(d)} = -\sum_{n \neq d} s_n^{(d)}$. We let $x_l^{(d)}$ be the amount of traffic on link l destined for node d, and call $x^{(d)} \in \mathbb{R}^L$ the *flow vector* for destination d. At each node n, the components of the flow vector and the source-sink vector for the same destination satisfy the conservation law

$$\sum_{l \in \mathcal{O}(n)} x_l^{(d)} - \sum_{l \in \mathcal{I}(n)} x_l^{(d)} = s_n^{(d)}, \quad d = 1, \dots, D,\ n = 1, \dots, N$$

The flow conservation law across the network can compactly be written as

$$Ax^{(d)} = s^{(d)}, \qquad d = 1, \dots, D$$

where A is the node-link incidence matrix defined above. Finally, we require that the total amount of traffic on link l, $\sum_d x_l^{(d)}$, does not exceed the link capacity c_l, *i.e.*, that

$$\sum_{d=1}^{D} x_l^{(d)} \leq c_l \qquad l = 1, \dots, L$$

For wireless networks, the capacities c_l depend on power allocation and media access schemes used, which we discuss in detail in Section 2.2.

In summary, our model imposes the following constraints on the variables $x^{(d)}$ and $s^{(d)}$:

$$\begin{array}{l} Ax^{(d)} = s^{(d)}, \quad x^{(d)} \succeq 0,\ s^{(d)} \succeq_d 0,\ d = 1, \dots, D \\ \sum_{d=1}^{D} x^{(d)} \preceq c \end{array} \tag{1}$$

where $\succeq$ denotes componentwise inequality, $\succeq_d$ means componentwise inequality except for the dth component, and c is the vector of link capacities. It should be noted that this model describes the average behavior of data transmissions (that is, the average data rates) and ignores packet-level details of transmission protocols and forwarding mechanisms.

Network flow model for fixed routing: It is sometimes natural to keep the routes between the source-destination pairs fixed, and only allow the source rates to vary. We then label the source-destination pairs by integers $p = 1, \dots, P$, and let s_p denote the data rate communicated between source-destination pair p. (It is also possible to have multiple routes per node pair.) In place of the node-link incidence matrix, we use the *link-route incidence matrix* $R \in \mathbb{R}^{L \times P}$ whose entries r_{lp} are defined via $r_{lp} = 1$ if the traffic between node pair p is routed across link l, and $r_{lp} = 0$ otherwise. The vector of total traffic across the links is given by Rs, and the fixed routing model imposes the following constraints on the end-to-end rates s,

$$Rs \preceq c, \qquad s \succeq 0 \tag{2}$$

B. Communications model

In a wireless system, the capacities of individual links depend on the media access scheme and the allocation of communications resources, such as transmit powers or time-slot fractions, to the transmitters. In this paper, we will consider a CDMA system where all the communication links share the same frequency band.

Radio propagation model: Let G_{lm} denote the effective power gain between the transmitter of link m and the receiver of link l. We use a deterministic fading model $G_{lm} = K_{lm} d_{lm}^{-\alpha}$ where d_{lm} is the distance between transmitter m and receiver l, α is a constant path loss exponent and K_{lm} is a normalization constant. The normalization constant depends on the radio propagation properties of the environment, but also allows us to account for the effects of coding gain, spreading gain, beamforming, etc. (see, *e.g.*, [3], [14])

Power control, transmission scheduling and link capacities: Let P_l be the power used by the transmitter of link l. We assume that each transmitter l is subject to a simple power limit $0 \leq P_l \leq P_{l,\max}$. We let σ_l denote the thermal noise power at its receiver, and define the signal to interference and noise ratio (SINR) of link l as

$$\gamma_l(P) = \frac{G_{ll} P_l}{\sigma_l + \sum_{m \neq l} G_{lm} P_m} \tag{3}$$

where $P = \begin{pmatrix} P_1 & \dots & P_L \end{pmatrix}$ is the vector of transmit powers.

We assume that data is coded separately for each link and that receivers do not decode third-party data, hence treat it as noise. Each link can then be viewed as a single-user Gaussian channel with Shannon capacity $c_l = W \log(1 + \gamma_l(P))$ where W is the system bandwidth. In practice, however, most communication schemes will achieve significantly lower rates, in particular when the coding block size is limited (see [15]). To be able to capture this effect, we will use the model

$$c_l = c_{\text{tgt},l}^{(r)} \qquad \text{if } \gamma_{\text{tgt},l}^{(r)} \leq \gamma_l(P) < \gamma_{\text{tgt},l}^{(r+1)} \tag{4}$$

with $c_{\text{tgt},l}^{(0)} = 0$, $\gamma_{\text{tgt},l}^{(0)} = 0$, and $\gamma_{\text{tgt},l}^{(r)} < \gamma_{\text{tgt},l}^{(r+1)}$. Here, $c_{\text{tgt},l}^{(r)}$ and $\gamma_{\text{tgt},l}^{(r)}$ denote the r^{th} discrete rate level and the associated SINR target, respectively. Thus, each transmitter may choose among a finite number of transmission rates depending on what SINR level it can sustain. This leads to a finite number of achievable *link-rate vectors*

$$c[k] = \begin{pmatrix} c_{\text{tgt}}^{(r_1)} & \cdots & c_{\text{tgt}}^{(r_L)} \end{pmatrix}$$

for some rate levels $(r_1 \quad \cdots \quad r_L)$, $k = 1, \dots, K$. Note that the number of link-rate vectors K may be exponential in the number of links L. By time-sharing over a large number of time slots, we can achieve the following polyhedral rate region

$$\mathcal{C} = \text{co}\Big\{ \Big(c_1^{(r_1)} \quad \cdots \quad c_L^{(r_L)}\Big) \;\Big|\; \gamma_l^{r_l} \le \gamma_l(P) < \gamma_l^{(r_l+1)} \text{ for some } P \text{ with } 0 \le P_l \le P_{l,\max} \Big\}$$

where co denotes the convex hull. We use the compact notation $c \in \mathcal{C}$ to denote that c is an achievable long-term average link rate, and note that any such point can be written as a convex combination of the discrete rate vectors,

$$c = \sum_{k=1}^{K} \alpha_k c[k], \quad \sum_{k=1}^{K} \alpha_k = 1, \quad \alpha_k \ge 0,\ k = 1, \dots, K$$

The scalars α_k represent the time fraction of the schedule that uses link-rate vector $c[k]$.

Transmission constraints: In many cases, technological constraints (*e.g.*, nodes equipped with omnidirectional antennas and no multi-user detection capabilities) limit nodes to communicate with at most one other node at a time. In this case, we let $\mathcal{C}$ be the convex hull of the discrete rate vectors that obey these transmission constraints.

C. Examples of rate and power adaption schemes

We will consider three particular classes of rate and power adaption schemes that are commonly used in the wireless networking literature. Further details of how they can be included in the optimization formulation are given in Section IV.

Scheme I: Fixed transmission rates and transmit powers: In this model, transmitters send with maximum power or stay silent. A collection of links can be active in the same time slot only if all active links exceed their signal-to-noise target $\gamma_{\text{tgt},l}$. The associated link rate is given by (4). Note that the rate on each active link is constant even if the link exceeds its SINR threshold. This is a common model for transmissions in 802.11, and has been used extensively in the literature (see, *e.g.*, [7], [9])

Scheme II: Fixed transmission rates and variable transmit powers: The use of maximum transmit powers in the above scheme is inefficient for two reasons. First, there is no increase in data rate once the SINR target is reached, so any power allocation that causes a link to exceed its SINR target consumes unnecessarily much energy. Secondly, high transmit powers lead to high interference levels and limit the number of links that can be activated at each time slot (cf. [13], [16]). In the second scheme that we consider, the transmitters of the active links adjust their transmit powers to increase spatial reuse.

Scheme III: Variable transmission rates and transmit powers: Wireless devices often support multiple data rates and mechanisms to switch between them are based on the channel conditions (see, *e.g.*, [17]–[20]). Such mechanisms have the potential to increase the data rates on links with good channel quality, hence increasing the total traffic that can be carried by the network. In the third scheme that we will consider, each link can choose between a finite set of transmission rates. Each transmission rate has an associated SINR target which must be met for the rate to be admissible.

III. The SRRAS Problem

Let $u_{\text{net}}(x, s)$ be a concave utility function of the link rates x and the end-to-end rates s. The simultaneously optimal routing, resource allocation and scheduling (SRRAS) can then be found by solving the following optimization problem

$$\begin{array}{ll} \text{maximize} & u_{\text{net}}(x, s) \\ \text{subject to} & Ax^{(d)} = s^{(d)},\ x^{(d)} \succeq 0,\ s^{(d)} \succeq_d 0,\ \forall\, d \\ & \sum_d x^{(d)} \preceq c \qquad c \in \mathcal{C} \end{array} \tag{5}$$

Note that this is a convex optimization problem, since we maximize a concave function subject to convex constraints. The SRRAS problem is very general and includes many important design problems for wireless networks. We conclude this section by detailing some of them.

Maximum throughput and transport capacity: One important performance metric for wireless data networks is the total throughput of the system. The combined routing, resource allocation and scheduling that gives the maximum weighted throughput can be put into the SRRAS form by considering

$$\begin{array}{ll} \text{maximize} & \sum_n \sum_{d \ne n} w_n^{(d)} s_n^{(d)} \\ \text{subject to} & \text{constaints in (5)} \end{array}$$

When all weights are equal, *e.g.* $w_n^{(d)} = 1$, this formulation reduces to total throughput maximization. Solving the problem for varying weights allows us to trace out the entire $D(N-1)$ dimensional capacity region.

The *transport capacity* [1] can be computed similarly, by replacing the objective function above by $\sum_{l=1}^{L} d_l \sum_{d=1}^{D} x_l^{(d)}$ where d_l is the physical length of link l (in meters).

Uniform capacity and maximizing the minimal rate: A related performance measure is the *uniform capacity*: the maximum aggregate communication rate when every node communicates with every other node in the network using a common rate (cf. [3]). The maximum common rate can be found by solving the following variant of SRRAS

$$\begin{array}{ll} \text{maximize} & \tau \\ \text{subject to} & s_n^{(d)} = \tau, \qquad \forall\, n \ne d, \quad \forall\, d \in \{1, \dots, N\} \\ & \text{and the constraints in (5)} \end{array}$$

The uniform capacity is obtained by multiplying the optimal value by $N(N-1)$.

Replacing the equalities $\tau = s_n^{(d)}$ by equalities $\tau \le s_n^{(d)}$ gives us a formulation for maximizing the minimal end-to-end rate that can be supported by the network. This is a basic building block in algorithms for computing *max-min* fair rates (see [21]).

Maximum utility SRRAS: As illustrated in [9], throughput maximization can lead to grossly unfair allocations of end-to-end communication rates. An alternative is to use a maximum-utility formulation as follows. Let $U_n^{(d)}(\cdot)$ be a concave and strictly increasing utility function, and let $U_n^{(d)}(s_n^{(d)})$ for $d \ne n$ represent the utility of node n for sending data at rate $s_n^{(d)}$ to destination d. Then, the maximum utility SRRAS problem is

$$\begin{array}{ll} \text{maximize} & \sum_n \sum_{d \ne n} U_n^{(d)}(s_n^{(d)}) \\ \text{subject to} & \text{constraints in (5)} \end{array}$$

This problem is closely related to fair allocation of end-to-end rates. Recall that a rate allocation $\bar{s}$ is called *proportionally fair* if, for all other feasible allocations s

$$\sum_n \sum_{d \neq n} \frac{\bar{s}_n^{(d)} - s_n^{(d)}}{s_n^{(d)}} \leq 0$$

It is well-known (see [22]) that s is proportionally fair if and only if it solves the above problem with $U_n^{(d)}(\cdot) = \log(\cdot)$.

IV. A Column Generation Approach to SRRAS

In this section, we will show how the SRRAS problem can be approached using a classical technique from mathematical programming known as column generation (cf. [7]). We will explain the technique on the SRRAS problem with fixed routing, and then extend the approach to the more general formulation (5). We conclude the section by deriving specific solution methods for the three rate and power adaption schemes from Section II-C.

A. Column generation for SRRAS with fixed routing

Consider the SRRAS problem with fixed routing

$$\begin{array}{ll} \text{maximize} & u(s) \\ \text{subject to} & Rs \preceq c, \quad s \succeq 0 \\ & c \in \mathcal{C} \end{array} \tag{6}$$

Here, $u(s)$ is a concave utility function, R is a given routing matrix and the optimization variables are s and c. Since $\mathcal{C}$ is a convex polytope, any element of $\mathcal{C}$ can be written as a convex combination of its extreme points $c[1], \ldots, c[K]$. This allows us to re-write (6) as the following optimization problem in s and α_k

$$\begin{array}{ll} \text{maximize} & u(s) \\ \text{subject to} & Rs \preceq c, \quad s \succeq 0 \\ & c = \sum_k \alpha_k c[k] \\ & \sum_k \alpha_k = 1, \qquad \alpha_k \geq 0, \quad k = 1, \ldots, K \end{array} \tag{7}$$

We refer to this problem as the *full master problem* and note that it is similar to the formulation used for investigating the capacity of a number of small ad hoc networks in [3]. Given the extreme points of $\mathcal{C}$, this is a convex optimization problem (maximizing a concave objective function subject to convex constraints). In general, however, this formulation is inconvenient for several reasons. Firstly, $\mathcal{C}$ may have a very large number of extreme points so explicit enumeration of all these quickly becomes intractable as the size of the network grows. Secondly, even when explicit enumeration is possible, the formulation (7) may have a very large (exponential) number of variables and can be computationally infeasible to solve directly. Rather than the brute force of enumerating all extreme points of $\mathcal{C}$, we will generate them "as we go along".

To this end, we consider a subset $\{c[k] \mid k \in \mathcal{K}\}$ of extreme points of $\mathcal{C}$, where $\mathcal{K} \subseteq \{1, \ldots, K\}$. The associated restriction of (7) to this subset is

$$\begin{array}{ll} \text{maximize} & u(s) \\ \text{subject to} & Rs \preceq c, \quad s \succeq 0 \\ & c = \sum_{k \in \mathcal{K}} \alpha_k c[k] \\ & \sum_{k \in \mathcal{K}} \alpha_k = 1, \qquad \alpha_k \geq 0 \quad k \in \mathcal{K} \end{array} \tag{8}$$

We will refer to (8) as the *restricted master problem*. Since this formulation optimizes over $c \in \mathcal{C}^{\mathcal{K}} = \text{co}\{c[k] \mid k \in \mathcal{K}\}$, and $\mathcal{C}^{\mathcal{K}} \subseteq \mathcal{C}$, this problem is a restriction of (7) and its optimal solution provides a lower bound u_{lower} to the optimal value u_{optimal} of the full master problem (7). The key idea of the column generation method is to sequentially improve the lower bound by adding new extreme points to the restricted master problem. Adding a new extreme point makes $\mathcal{C}^{\mathcal{K}}$ a better approximation of $\mathcal{C}$ and allows u_{lower} to approach u_{optimal}.

The key machinery for updating $\mathcal{C}^{\mathcal{K}}$ is Lagrange duality. We introduce Lagrange multiplier λ for the capacity constraint $Rs \preceq c$, and form the corresponding Lagrangian

$$L(s, c, \lambda) = u(s) - \lambda^T Rs + \lambda^T c.$$

In our implementation, we solve the restricted master problem to optimality using a primal-dual interior-point method. Let $(s^\star, \alpha^\star)$ be an optimal solution to (8), define $c^\star = \sum_{k \in \mathcal{K}} \alpha_k^\star c[k]$, and let $\lambda^\star$ denote the optimal Lagrange multipliers for the capacity constraints $Rs \preceq c$. Since the constraints in (8) are affine, feasibility of the restricted master problem implies strong duality [23, §5.2.3], and that u_{lower} can be expressed as

$$\begin{aligned} u_{\text{lower}} &= L(s^\star, c^\star, \lambda^\star) = \\ &= \sup_{s \succeq 0} \left\{ u(s) - \lambda^{\star T} Rs \right\} + \sup_{c \in \mathcal{C}^{\mathcal{K}}} \left\{ \lambda^{\star T} c \right\}. \end{aligned} \tag{9}$$

We can estimate how far u_{lower} is from optimality by generating a corresponding upper bound on u_{optimal}. This can be done by considering the corresponding dual formulation of the original problem (6). More specifically, by weak duality, for any $\lambda \succeq 0$, the value

$$g(\lambda) = \sup_{s \succeq 0,\, c \in \mathcal{C}} L(s, c, \lambda) = \sup_{s \succeq 0} \left\{ u(s) - \lambda^T Rs \right\} + \sup_{c \in \mathcal{C}} \left\{ \lambda^T c \right\}$$

provides an upper bound to u_{optimal}. In particular, we consider the bound given by $\lambda^\star$

$$u_{\text{upper}} = \sup_{s \succeq 0} \left\{ u(s) - \lambda^{\star T} Rs \right\} + \sup_{c \in \mathcal{C}} \left\{ \lambda^{\star T} c \right\}. \tag{10}$$

The difference $u_{\text{upper}} - u_{\text{lower}}$ serves as a measure of accuracy of the current solution. We consider $(s^\star, c^\star)$ to be the optimal solution to (6) if the difference drops below a predefined threshold. If the current solution does not satisfy the stopping criterion, we conclude that $\mathcal{C}$ is not sufficiently well characterized by the vertices $\{c[k]\}_{k \in \mathcal{K}}$ and that a new extreme point should be added to the description before the procedure is repeated.

When adding a new extreme point, we would like to use a vertex of $\mathcal{C}$ that allows u_{lower} to improve as much as possible. Note that $\lambda^\star$ gives the sensitivity of u_{lower} at $c^\star$. In other words, if we view u_{lower} as a function of c, then $\lambda^\star$ is a subgradient of u_{lower} at $c^\star$ [23, §5.6]; see Figure 1. It is thus natural to add the extreme point that solves

$$\begin{array}{ll} \text{maximize} & \lambda^{\star T} c \\ \text{subject to} & c \in \mathcal{C} \end{array} \tag{11}$$

We will call this problem the *scheduling subproblem* in our column generation method.

The column generation algorithm can now be summarized as follows. Given an initial set of vertices of $\mathcal{C}$, we solve

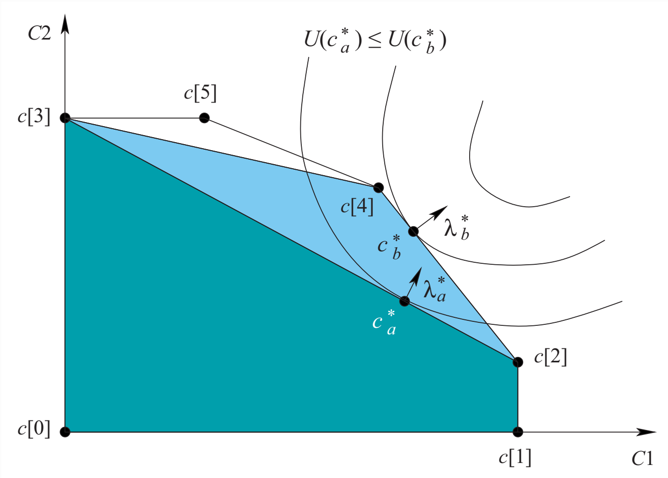

Fig. 1. Three level curves of $U(c) = \sup_s\{u(s) \mid Rs \preceq c,\ s \succeq 0\}$ are shown for a two-user case. Suppose that at certain iteration we have already generated extreme points $c[0], c[1], c[2], c[3]$, *i.e.*, that $\mathcal{C}^{\mathcal{K}}$ is the dark polytope. Solving the restricted master problem (8) gives the optimal capacity $c_a^\star$ and the corresponding Lagrange multiplier $\lambda_a^\star$. Then solving the scheduling subproblem (11) generates the (new) vertex $c[4]$. In the next iteration, solving the restricted master problem (8) (with $c[4]$ added) gives the optimal point $c_b^\star$ and the corresponding Lagrange multiplier $\lambda_b^\star$. Now solving the scheduling subproblem (11) with $\lambda_b^\star$ generates $c[2]$ and/or $c[4]$, which are both already in the generated list. We thus conclude that $c_b^\star$ is the global optimal point. Note that the true capacity region $\mathcal{C}$ is the convex polytope with extreme points $c[0], c[1], \ldots, c[5]$, and we never generated $c[5]$ in the column generation procedure. For larger networks, the method typically only generates a small fraction of the vertices of the capacity region; cf. Section V

the restricted master problem to get a lower bound on the performance and the optimal Lagrange multipliers for the capacity constraint. To evaluate the upper bound, we solve a network flow subproblem $\sup_{s\succeq 0}\left\{u(s) - \lambda^{\star T} Rs\right\}$ and the scheduling subproblem (11). If the bounds are sufficiently close, we terminate the algorithm. Otherwise, we add the vertex generated by solving the subproblem and start the next iteration; see the flow chart in Figure 2. Note that both the restricted master problem and the network flow subproblem can be solved efficiently using standard methods for convex optimization. The computationally most expensive part of each iteration is solving the subproblem (11), which is often formulated as a mixed-integer linear program (see §IV-C).

To establish convergence of the algorithm, note that if a new extreme point is generated at each iteration, then after a finite number (less than K) of steps all extreme points are generated, $\mathcal{C}^{\mathcal{K}} = \mathcal{C}$, and hence $u_{\text{lower}} = u_{\text{upper}}$. However, it often happens that within a relatively small number of steps an old extreme point is generated, *i.e.*, the optimal solution to (11) is already in $\mathcal{C}^{\mathcal{K}}$. This implies that

$$\sup_{c\in\mathcal{C}} (\lambda^\star)^T c = \sup_{c\in\mathcal{C}^{\mathcal{K}}} (\lambda^\star)^T c. \tag{12}$$

By comparing equations (10) and (9), we conclude that this also implies that $u_{\text{lower}} = u_{\text{optimal}} = u_{\text{upper}}$, and that the optimal solution to the restricted master problem is also optimal to the full master problem. Thus, we have solved the original problem without generating all the extreme points; cf. Figure 1.

Column generation: We can understand the name "column generation" by introducing the matrix $C^{\mathcal{K}}$ whose columns are the link-rate vectors $\{c[k] \mid k \in \mathcal{K}\}$, and writing the capacity constraint of the restricted master problem as

$$Rs \preceq C^{\mathcal{K}}\alpha$$

Adding an extreme point to $\mathcal{C}^{\mathcal{K}}$ is then equivalent to generating a new column in $C^{\mathcal{K}}$.

B. Column generation for the general SRRAS problem

The column generation method is directly applicable to the SRRAS problem (5). In this case, we compute a lower bound u_{lower} by solving

$$\begin{array}{ll} \text{maximize} & u_{\text{net}}(x,s) \\ \text{subject to} & Ax^{(d)} = s^{(d)}, \quad x^{(d)} \succeq 0, \quad s^{(d)} \succeq_d 0, \quad \forall d \\ & \sum_d x^{(d)} \preceq c \\ & c = \sum_{k\in\mathcal{K}} \alpha_k c[k] \\ & \sum_k \alpha_k = 1, \qquad \alpha_k \ge 0, \quad k \in \mathcal{K} \end{array}$$

while the upper bound is computed as

$$u_{\text{upper}} = \sup_{\substack{x^{(d)}\succeq 0\\ s^{(d)}\succeq_d 0}} \left\{ u_{\text{net}}(x,s) - \sum_d \lambda^T x^{(d)} \mid Ax^{(d)} = s^{(d)} \right\} + \sup_{c\in\mathcal{C}} \left\{\lambda^T c\right\}$$

Note that in computing the upper bound, the first part requires the solution of an uncapacitated network flow problem, while the second subproblem is identical to (11) which appeared in the fixed-routing formulation. In all other respects, the column generation procedure proceeds as for the fixed-routing case.

C. Generating feasible link rate vectors

The nature of the feasible rate region and, hence, of the scheduling subproblem (11) depends on the rate and power adaption scheme that we employ. We would like to stress that the column generation method can be applied to *any* discrete rate/power adaption scheme for which we can solve the associated weighted maximum-throughput problem (11). In this section, we will only demonstrate how the subproblem corresponding to the schemes outlined in Section II-C can be formulated as mixed-integer linear programs on standard form and, thus, solved numerically using publically available software. For sake of readability, we assume that the power limits and SINR thresholds are equal for all links.

Scheme I: Fixed transmission rates and transmit powers: In this scheme, a collection of links can transmit data simultaneously if their signal to interference and noise ratios exceed their target values. In other words, active links must satisfy

$$G_{ll}P_l \ge \gamma_{\text{tgt}} \left(\sigma_l + \sum_{j\neq l} G_{lj}P_j \right) \tag{13}$$

Active transmitters use their maximal power $P_{\max}$ and transmit at rate c_{tgt}. To express this condition in a mathematical programming framework, introduce the boolean variables

$$x_l = \begin{cases} 1 & \text{if sender } l \text{ is transmitting} \\ 0 & \text{otherwise} \end{cases}$$

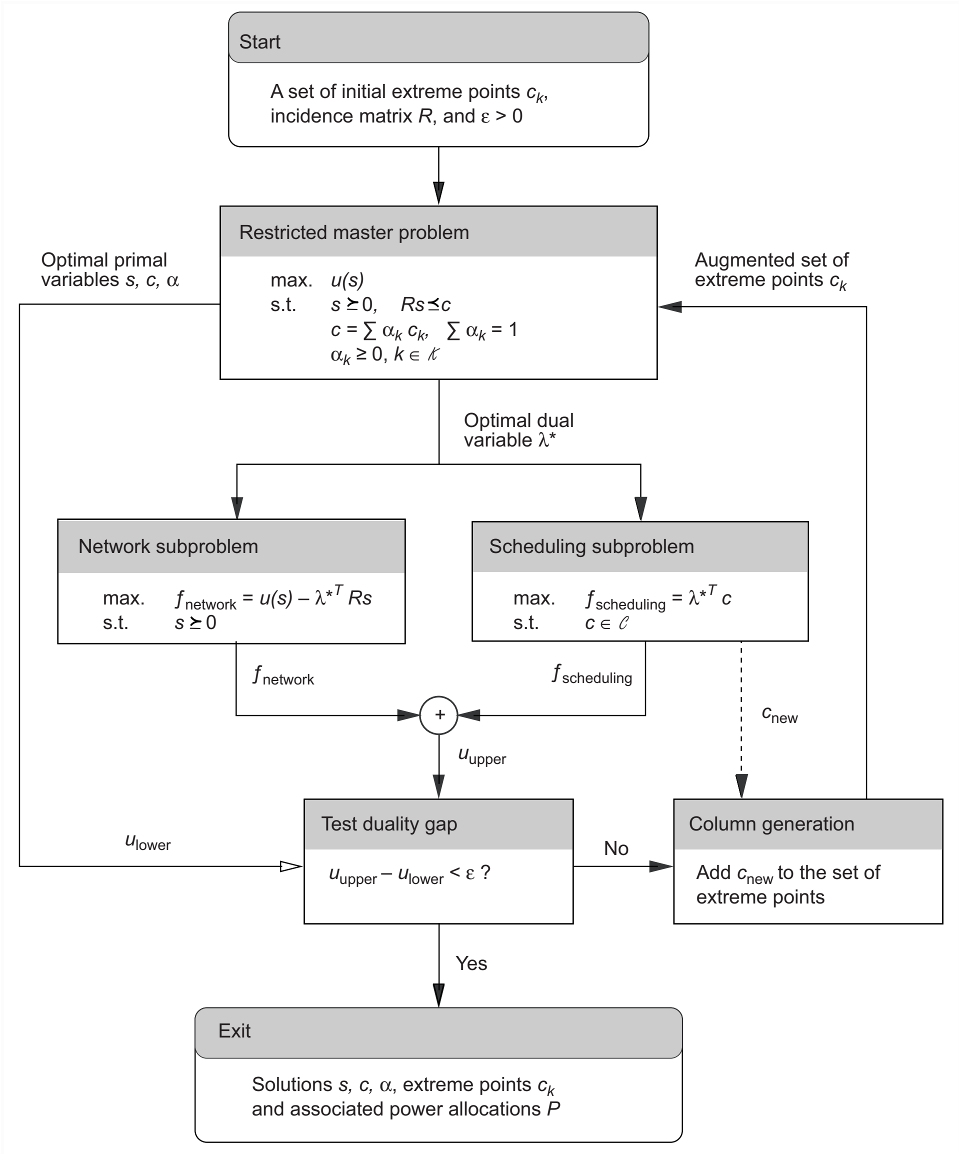

Fig. 2. Flow chart for the column generation method.

We can now write the interference constraints as

$$G_{ll}P_{\max}x_l + M_l(1-x_l) \geq \gamma_{\text{tgt}}\left(\sigma_l + \sum_{j\neq l} G_{lj}P_{\max}x_j\right)$$

where M_l is a sufficiently large constant. We will use the value $M_l = \gamma_{\text{tgt}}\left(\sigma_l + \sum_{j\neq l} G_{lj}P_{\max}\right) + G_{ll}P_{\max}$ which results in the transmission constraints

$$\begin{aligned}&\left(\sigma_l + \textstyle\sum_{j\neq l} G_{lj}P_{\max}\right)x_l + \textstyle\sum_{j\neq l} G_{lj}P_{\max}x_j \\ &\leq \textstyle\sum_{j\neq l} G_{lj}P_{\max} + \gamma_{\text{tgt}}^{-1}G_{ll}P_{\max}\end{aligned} \tag{14}$$

Thus, we suggest to generate transmission groups by solving the subproblems

$$\begin{array}{ll}\text{maximize} & \lambda^T x \\ \text{subject to} & (14), \quad x_l \in \{0,1\}, \quad l = 1,\dots,L\end{array} \tag{15}$$

which is a mixed integer-linear program on standard form.

Scheme II: Fixed transmission rates and variable transmit powers: Combined scheduling and power control can be performed similarly, by re-writing (13) as

$$G_{ll}P_l + (1-x_l)M_l \geq \gamma_{\text{tgt}}(\sigma_l + \sum_{j\neq l} G_{lj}P_j)$$

for a sufficiently large constant M_l. Using the value $M_l = \gamma_{\text{tgt}}\left(\sigma_l + \sum_{j\neq l} G_{lj}P_{\max}\right)$, we find that the transmission constraints can be re-written as

$$\begin{aligned}&-G_{ll}P_l + \gamma_{\text{tgt}}\textstyle\sum_{j\neq l} G_{lj}P_j + \\ &\gamma_{\text{tgt}}\left(\sigma_l + P_{\max}\textstyle\sum_{j\neq l} G_{lj}\right)x_l \\ &\leq \gamma_{\text{tgt}}P_{\max}\textstyle\sum_{j\neq l} G_{lj}\end{aligned} \tag{16}$$

Maximizing $\lambda^T x$ over these constraints, *i.e.*, solving

$$\begin{array}{lll}\text{maximize} & \lambda^T x & \\ \text{subject to} & (16) & l = 1,\dots,L \\ & 0 \leq P_l \leq P_{\max},\ x_l \in \{0,1\} & l = 1,\dots,L\end{array} \tag{17}$$

finds the most advantageous group of transmitters that can be active during a time slot. Since there may be many combinations of transmit powers that allow the links in the optimal transmission groups to meet their SINR targets, we can apply the classical power control algorithm (*e.g.*, [24]) to find the allocation of minimum total power that admits simultaneous activation of the links.

Scheme III: Variable transmission rates and powers: The above approach is easily extended to the situation where nodes can transmit at a finite set of rates, depending on the achievable SINR levels. We assume that link l can transmit at rate $c^{(r)}$ if

$$G_{ll}P_l \geq \gamma_{\text{tgt}}^{(r)} \left(\sigma_l + \sum_{j \neq l} G_{lj} P_j \right)$$

Introducing the boolean variables $x_l^{(r)} = 1$ if link l transmits at rate r, and $x_l^{(r)} = 0$ otherwise we can re-write the transmission constraint as

$$\begin{aligned} &-G_{ll}P_l + \gamma_{\text{tgt}}^{(r)} \textstyle\sum_{j \neq l} G_{lj} P_j \\ &+\gamma_{\text{tgt}}^{(r)} \left(\sigma_l + P_{\max} \textstyle\sum_{j \neq l} G_{lj} \right) x_l^{(r)} \\ &\leq \gamma_{\text{tgt}}^{(r)} P_{\max} \textstyle\sum_{j \neq l} G_{lj} \end{aligned} \tag{18}$$

Since each link can only transmit at a single rate, we also require that

$$\sum_r x_l^{(r)} \leq 1 \qquad l = 1, \ldots, L \tag{19}$$

Finally, we will have to account for the fact that links can transmit at different rates when solving the subproblem. In summary, we propose to solve

$$\begin{array}{lll} \text{maximize} & \sum_l \lambda_l \sum_r c_{\text{tgt}}^{(r)} x_l^{(r)} & \\ \text{subject to} & (18), (19) & \forall\, l,\ \forall\, r \\ & 0 \leq P_l \leq P_{\max},\ x_l \in \{0, 1\} & \forall\, l \end{array} \tag{20}$$

Transmission constraints: In cases where nodes are limited to only send or receive data on one link at a time, we have to include the additional linear constraint

$$\sum_{l \in \mathcal{O}(n)} x_l + \sum_{m \in \mathcal{I}(n)} x_m \leq 1 \qquad n = 1, \ldots, N.$$

in the subproblems (15), (17) and (20).

V. Examples

In this section, we use our approach to gain some insight into how power control, spatial reuse, routing strategies and variable transmission rates influence the network performance. We will consider the case where every node has data to send to other nodes, perhaps by multihop routing. As the results depend on both the traffic situation and the parameters of the radio link model, they should be seen as an indication of what type of questions that can be addressed using this framework rather than facts about wireless network performance.

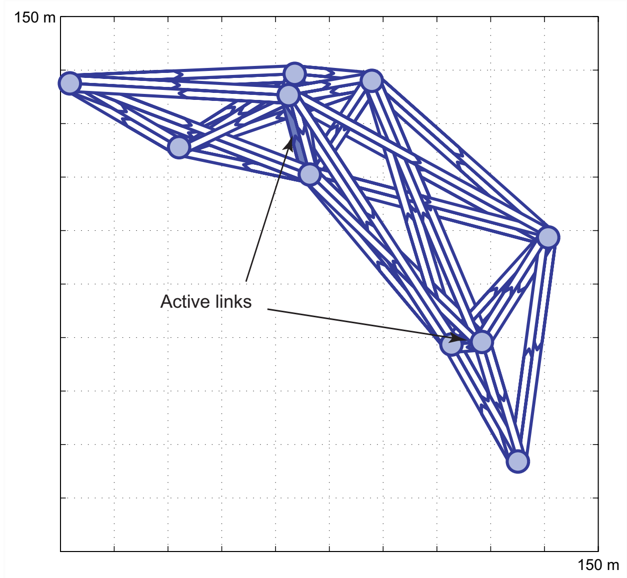

Fig. 3. Topology of sample network and the two active links for throughput-optimal solution under maximum power transmissions.

A. Generating sample network scenarios

To evaluate our methodology, we construct a set of sample networks using a radio link model that broadly corresponds to a hypothetical high-speed indoor wireless LAN using the entire 2.4000–2.4835 GHz ISM band (see [25] for details). We let $P_{\max} = 100\text{mW}$, $\alpha = 3$, $K_{lm} = 2 \times 10^{-4}$, $\sigma = 3.34 \times 10^{-12}$. We use the Shannon capacity formula $c_{\text{tgt}}^{(r)} = W \log_2(1 + \gamma_{\text{tgt}}^{(r)})$ to relate target SINR-levels to rates. Using $W = 83.5 \times 10^6$, and a SNIR-target $\gamma_{\text{tgt}} = 10$, we find the base rate 288.9 Mbps. For multiple-rate scenarios, we will assume that the system can also offer half and double this rate (with associated SINR targets of 3.46 and 120, respectively). To generate the network topology, we place nodes randomly on a square and introduce links between every pair of nodes that can sustain the base target SINR when all other transmitters are silent (in our model, this corresponds to a distance of 84.2 m). We then adjust the dimension of the square so that the nodes form a network that is fully connected (*i.e.*, that there is at least one path between any pair of nodes) and that $L/N(N-1)$ (*i.e.*, the average number of node pairs that are connected by direct links) matches a desired target number.

B. Performance objectives for wireless network optimization

Our first investigation considers the adequacy of various performance objectives in the network optimization. We present specific results for the network shown in Figure 3, but have found qualitatively similar results for all scenarios that we have considered.

Generally speaking, we have found throughput maximization to be an inappropriate objective in the optimization. Maximum throughput solutions tend to activate a few (typically short) links and allocates non-zero rates to the flows that only traverse these links. All other flows are set to zero. The throughput-optimal solution for the network in Figure 3, for example, activates only the two highlighted links.

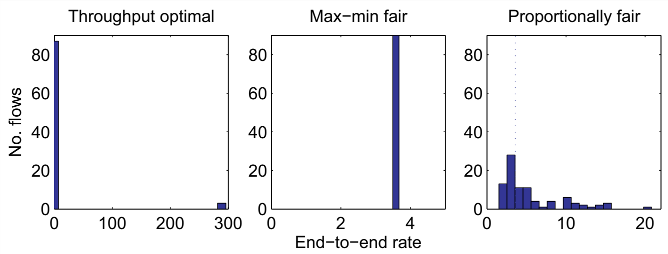

Fig. 4. Flow distributions for throughput optimization (left), equal-rate allocation (middle) and proportionally fair solution (right). The results are for the network in Figure 3 under free routing and transmission scheme II.

The problem can be avoided by optimizing with respect to logarithmic utility (proportional fairness) or equal end-to-end rate assignment (uniform capacity). The distribution of flow rates for the different approaches are shown in Figure 4. As one can see, both these approaches allocate non-zero rates to all flows; the proportionally fair solution can allocate relatively large rates to some flows at the expense of a slight decrease in the rates for a few small flows. The equal-rate allocation attains 37.2% of the achievable throughput while the proportional fair solution results in a throughput of 57.8% of the maximum achievable. The results have been qualitatively similar for a large number of networks that we have considered: the equal rate allocation results in a large decrease in total throughput, while the proportionally fair allocation makes a more balanced trade-off between throughput and fairness. These observations are consistent with the findings in [9].

C. The influence of routing and rate/power adaption schemes

Next, we try to quantify the benefits of flexible routing and rate/power adaption schemes on our sample networks. We focus on the fair rate allocation problems, and start out by analyzing the solutions for the network in Figure 3. Table I shows the maximum equal rate allocations that can be achieved using various rate/power schemes under fixed (shortest-path) and free routing. The entry "reuse" gives the average number of links that are active in each time-slot, while "transport efficiency" is transport capacity divided by average transmit power. To get a fair comparison for variable power schemes, we apply the classical power control algorithm (see, *e.g.*, [24]) to allocate transmit powers to the active links in each slot.

As we can see, variable power transmissions and variable rate selection give throughput increases of 24.2% and 31.1%, respectively. The variable power transmissions give a good increase in the average reuse factor, with a somewhat smaller increase for the variable-rate adaption. The "transport efficiency" is increased by 22.7% when variable power is introduced, but then decreased under variable rate transmissions. This is due to the large increase in power necessary for sustaining the higher transmission rates. As can be seen in Table I, the influence of flexible routing scheme is quite significant for this network: combined variable-rate transmissions and free routing results in a performance increase of 66.4% over maximum power transmissions and shortest-hop routing. The corresponding results for proportionally fair rate allocation under free routing

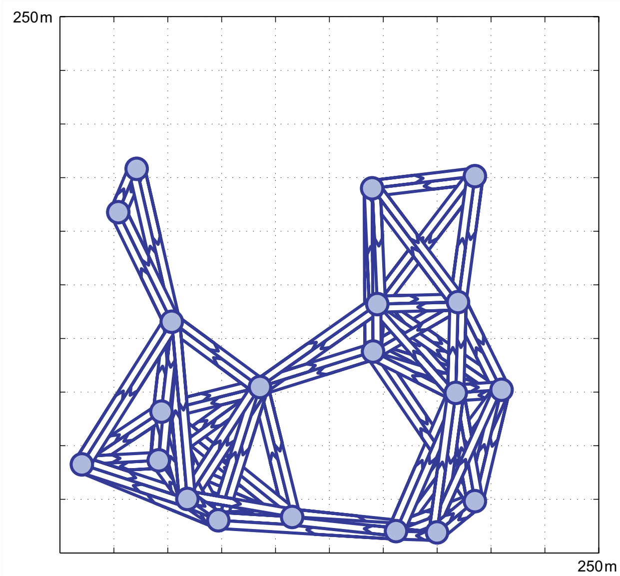

Fig. 5. Additional network scenarios with 20 nodes.

are shown in Table II. A substantial increase in throughput compared to the equal-rate assignment has been achieved at the expense of a relatively slight decrease in the smaller rates. Note that the general results of this section are quite different from the findings in [3], where only very small improvements where obtained with variable power transmissions.

We have done the same investigations for a large number of networks. For a set of 60 sample networks, the performance increases from moving from maximum power transmissions to variable transmit power gave performance increases of $0.5\% - 74.3\%$, with an average around 22.9%. For the larger 20-node network shown in Figure 5 we notice that the routing has a strong influence: the equal-rate allocation under variable power transmissions is increased by 50% when we move from shortest-hop to free routing, and the reuse factor can be increased from 2.15 to 3.57.

D. Fairness-throughput regions

As we have seen in Section V-B, there is a clear trade-off between throughput and fairness in wireless networks. Optimizing throughput typically results in a few short flows getting all the network resources while the majority of flows are not allowed to transmit at all. Fair rate allocations, on the other hand, result in significantly decreased throughput. In this section, we will try to shed some light on this tradeoff by solving the family of problems

$$\begin{array}{ll} \text{maximize} & \beta \sum_n \sum_{d\neq n} s_n^{(d)} + (1-\beta) \sum_n \sum_{d\neq n} U_n^{(d)}(s_n^{(d)}) \\ \text{subject to} & \text{constraints in } (5) \end{array}$$

Note that this problem reduces to throughput maximization when $\beta = 1$, and to utility maximization when $\beta = 0$. The solutions to this problem are *Pareto optimal*, in the sense that an increase in throughput must come at the expense of a decrease in log-utility and vice versa (see [23]). By solving the problem for all $\beta \in [0, 1]$, we can trace out the achievable combinations of throughput and total log-utility. The resulting trade-off curves for the network in Figure 5 are shown in

TABLE I

EQUAL-RATE ALLOCATIONS FOR NETWORK IN FIGURE 3 UNDER FREE ROUTING (TOP) AND SHORTEST-HOP ROUTING (BOTTOM).

	Throughput	End-to-end rate	Reuse	Transport efficiency
Fixed rate/fixed power	250.0	2.78	1.30	$135 \cdot 10^3$
Fixed rate/variable power	322.3	3.58	2.50	$265 \cdot 10^3$
Variable rate/variable power	366.5	4.07	2.16	$247 \cdot 10^3$

	Throughput	End-to-end rate	Reuse	Transport efficiency
Fixed rate/fixed power	220.3	2.45	1.28	$186 \cdot 10^3$
Fixed rate/variable power	273.7	3.04	2.31	$228 \cdot 10^3$
Variable rate/variable power	288.7	3.21	1.94	$203 \cdot 10^3$

TABLE II

PROPORTIONALLY FAIR ALLOCATIONS FOR NETWORK IN FIGURE 3 UNDER FREE ROUTING. THE END-TO-END RATES COLUMN GIVE THE MINIMAL, MEDIAN AND MAXIMAL RATES, RESPECTIVELY.

	Throughput	End-to-end rates	Reuse	Transport efficiency
Fixed rate/fixed power	349.8	[1.38, 3.21, 9.81]	1.52	$100 \cdot 10^3$
Fixed rate/variable power	501.0	[1.88, 3.98, 20.31]	2.42	$309 \cdot 10^3$
Variable rate/variable power	639.1	[1.97, 4.44, 46.42]	2.34	$320 \cdot 10^3$

Figure 6. As can be expected, the benefits of routing on the throughput when we can accept small values of log-utility is quite marginal. There is, however, a clear benefit of both variable power and variable rate selection.

It is interesting to compare these results with what can be achieved by time-sharing between the throughput-optimal and the proportionally fair allocation. Such an approach would, roughly speaking, augment a proportionally fair schedule with some time-slots in which a few high-quality links could transmit at high rate. Although this approach is not optimal, plotting the resulting utility-throughput combinations and comparing this with the Pareto optimal surface reveals that the two curves are relatively close; see Figure 6(bottom).

E. Computational experience

The approach described in this paper has exponential complexity in the number of links. This has forced us to limit our investigations to networks with less than 20 nodes, with computation times ranging from a few seconds to several hours. Although much work could be done on improving the subproblem solvers, these problems are combinatorial in nature and computing exact solutions is, in general, NP-hard. In particular, even the simplified problem of scheduling maximum power transmissions (15) while disregarding transmission constraints is a multiconstraint knapsack problem, which is known to be NP-hard in general (see, *e.g.*, [26]). Despite such negative results, the practical performance of mixed-integer linear programming solvers has allowed us to investigate a large number of non-trivial networks with a reasonable computational effort. To get a feel for how restrictive the transmission and interference constraints are, consider again the network in Figure 3. This network has $2^{56} \approx 7.2 \cdot 10^{16}$ possible combinations of links that can be active in a time-slot. Only 19253 of these satisfy the transmission constraints. Maximum power transmissions restricts the number of feasible link activations to a mere 178, while variable power transmissions make 16617 of these feasible. The column generation method itself has finite convergence, with a distinct "homing in/tailing off"-behavior, where initial progress is very steep but the convergence to a high precision solution is relatively slow; see Figure 7. For the ten-node networks that we have studied, our algorithm converges in $30 - 70$ iterations.

VI. CONCLUSIONS

We have considered the problem of finding the optimal end-to-end rate selection, routing, power allocation and transmission scheduling for wireless networks. Our objective has been to optimize throughput and (proportional) fairness in the network. We have shown how realistic models of power and rate adaption schemes can be incorporated in our model, and how the resulting optimization problem can be formulated as a nonlinear optimization problem. For a given network configuration, our approach provides the optimal operation of transport, routing and radio link layers under several important power and rate adaption schemes, as well as the optimal coordination across layers. This allows us to gain insight in the influence of power control, spatial reuse, routing strategies and variable transmission rates on the network performance, and provides a benchmark for alternative (heuristic) strategies. We have developed a specialized solution method based on Lagrange duality and column generation and demonstrated the approach on several examples. The computational aspects of the algorithm can be improved in many respects. This includes better choice of Lagrange multipliers in the column generation and faster methods for solving the column generation subproblem (see, *e.g.*, [16], [18]). We are currently investigating how this work can be extended to other rate and power adaption schemes, and how the centralized algorithm can be translated into distributed protocols.

ACKNOWLEDGEMENTS

The first author is grateful for fruitful discussions with the Radio Systems Group at KTH and, in particular, Dr. Tim Giles

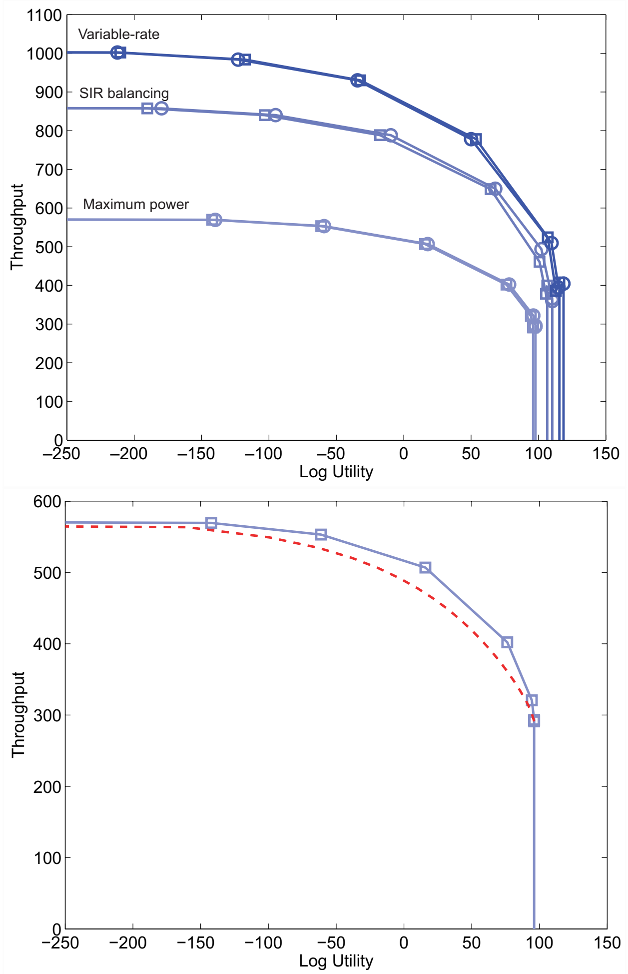

Fig. 6. Achievable combinations of log-utility and throughput for different rate/power and routing schemes (top). The bottom figure shows a comparison of the Pareto optimal solutions (full) and a simple time-sharing between throughput optimal and proportionally fair allocations (dashed).

for devising the network scenario.

APPENDIX

A. Convergence of the column generation approach

First of all note that Slater's condition, and thus strong duality, holds. Thus, there is no duality gap. Let $(s^\star, \alpha^\star)$ be an optimal solution to the restricted master problem, and let $\lambda^\star$ be the associated Lagrange multipliers for the capacity constraints $Rs \preceq C\alpha$. Then,

$$\begin{aligned} u_{\text{lower}} &= u(s^\star) - (\lambda^\star)^T(Rs - C\alpha) = \\ &= \max_{s \succeq 0} \left\{ u(s) - (\lambda^\star)^T Rs \right\} + \max_{\alpha \succeq 0, \mathbf{1}^T\alpha = 1} \left\{ (\lambda^\star)^T C\alpha \right\} = \\ &= \max_{s \succeq 0} \left\{ u(s) - (\lambda^\star)^T Rs \right\} + \max_{c \in \mathcal{C}^{\mathcal{K}}} (\lambda^\star)^T c \leq u_{\text{optimal}} \end{aligned}$$

On the other hand, if we evaluate the upper bound for $\lambda = \lambda^\star$, we find

$$u_{\text{optimal}} \leq \max_{s \succeq 0} \left\{ u(s) - (\lambda^\star)^T Rs \right\} + \max_{c \in \mathcal{C}} (\lambda^\star)^T c$$

Thus, if

$$\max_{c \in \mathcal{C}} (\lambda^\star)^T c = \max_{c \in \mathcal{C}^{\mathcal{K}}} (\lambda^\star)^T c \tag{21}$$

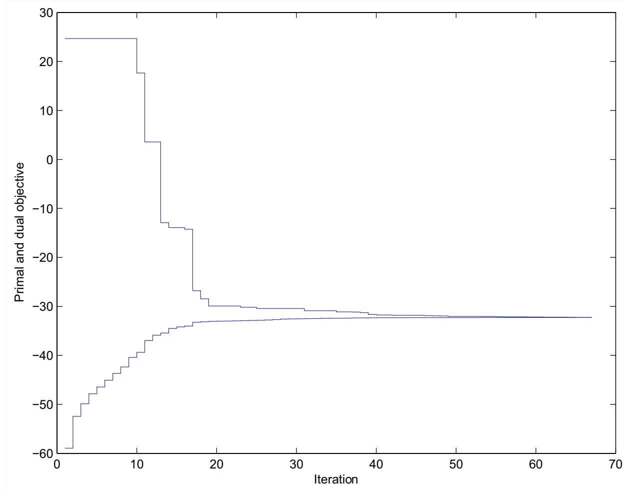

Fig. 7. Primal and dual values as function of iteration number; the solution shows a "homing-in, tailing-off" behavior where initial progress is steep, but many iterations are required to reach the optimal solution.

then the upper and lower bounds are equal, and the algorithm terminates having found an optimal solution. In particular, note that if the subproblem returns a vertex already in $\mathcal{C}^{\mathcal{K}}$, then (21) holds and the algorithm terminates with an optimal solution.

To verify that Slater's condition holds, note that if the network is connected then data can be routed between every pair of nodes and there is a strictly feasible solution. One such solution is obtained by combining TDMA transmissions (*i.e*, use $c[1], \ldots, c[L]$ defined via $c_k[k] = c_{\text{tgt}}$ and $c_l[k] = 0$ for $l \neq k$) under half the schedule length ($\sum_{k=1}^{L} \alpha_k = 0.5$) with zero transmissions under the rest of the schedule ($c[L+1] = 0,\ \alpha_{L+1} = 0.5$). The resulting link rate vector $\tilde{c}$ is strictly positive and lies in the relative interior of $\mathcal{C}$. An associated strictly feasible end-to-end rate vector $\tilde{s}$ can be constructed by assigning the (strictly positive) rate $\min_l \left\{ \tilde{c}_l / \sum_p r_{lp} \right\}$ to all node pairs. This allocation ensures that $R\tilde{s} \preceq \tilde{c}/2$. Thus $(\tilde{s}, \tilde{c})$ constructed in this way is strictly feasible.

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